

*Coarse Grained Model of a Nanoparticle Inclusion in a Cell's Membrane*

Giovanni Marelli

Università degli studi di Padova

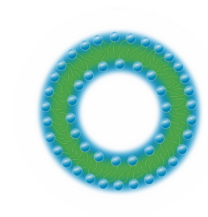
March 11, 2009



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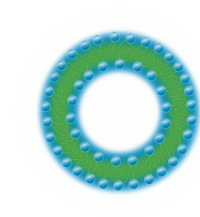
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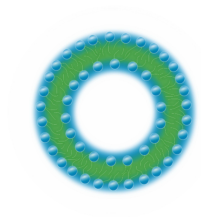
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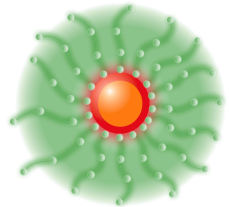


The nanoparticles are particles ranging in the nanometer scale. They are composed by a metallic (Au) or semiconductor core (CdSe, ZnS), they can have zinc coating, and an external layer of grafted polymers. The polymer brush provides the degrees of polarity of the nanoparticles.

The nanoparticles can be inserted into the vesicles and interact with external electric field. Semiconductor nanoparticles have particular fluorescent spectral lines and trace the position of the liposome and polymersome. Gold nanoparticles can resonate under a tuned electrical stimulus, heat up and kill the malignant cells.

It is of fundamental importance the stability of the nanoparticle in the membrane. The nanoparticles should move together with the vesicles and trace their positions.

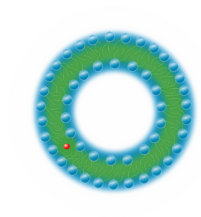
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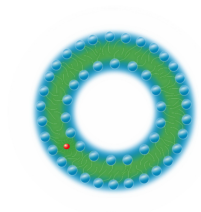
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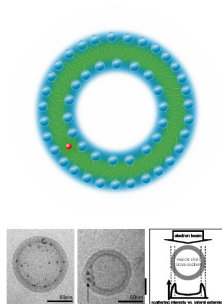
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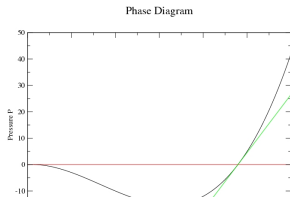
An hydrophobic polymer chain in a polar solvent minimize the surface area with the surrounding. The third-order virial expansion is the lowest order equation of state that allows the phase coexistence of the melt with its vapour. The model is coarse-grained and solvent free and provides on-site the effective interactions between the chains. The determination of the virial coefficients is provided by the incompressibility and the coexistence density of the melt. The density profile of the melt shows a segregation of the melt in a portion of space where the density is constant  $\rho_{coex}$ . The liquid-vapour interface is sharp.

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$$k_T^{-1} = \rho + v_{AA} \rho^2 + 2v_{AAA} \rho^3$$

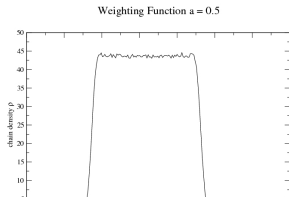


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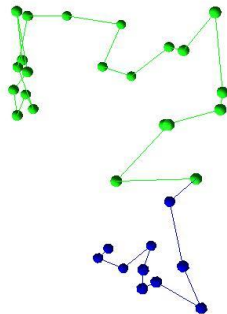


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The new species adds new interactions. Estimation of the new virial coefficients.

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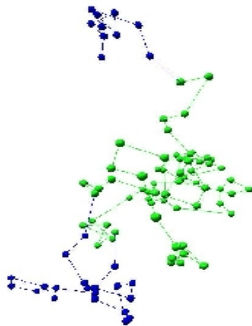


Flory-Huggins

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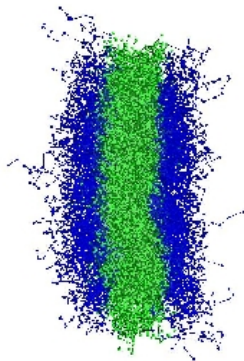


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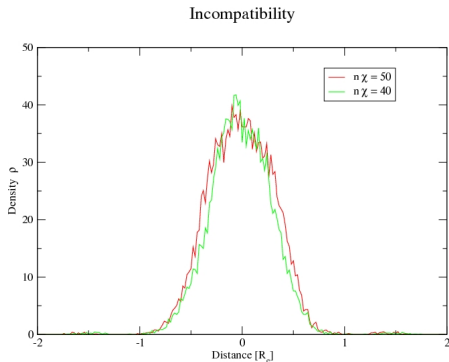
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We can control the thickness,  $d_t$ , and the density of chains per area  $\sigma_c$ , varying the number density of chains,  $\rho_c$ , and the incompatibility between the two species  $\chi N$ .



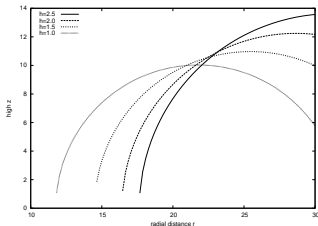
$\chi N$	$d_t$ [ $R_e$ ]	$S_c$ [ $R_e^2$ ]
30	0.826	0.0197
40	0.834	0.0186
50	0.92	0.0150

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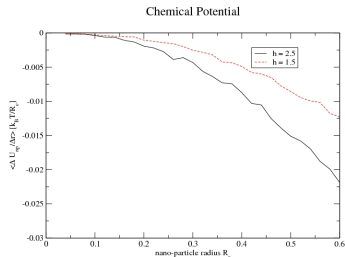
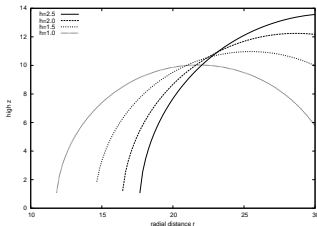
The Hamaker constant controls the contact angle with the hydrophylic melt



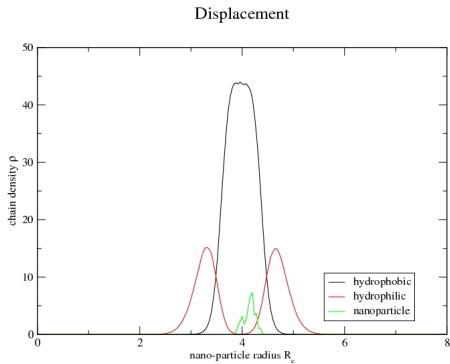
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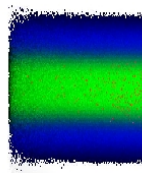
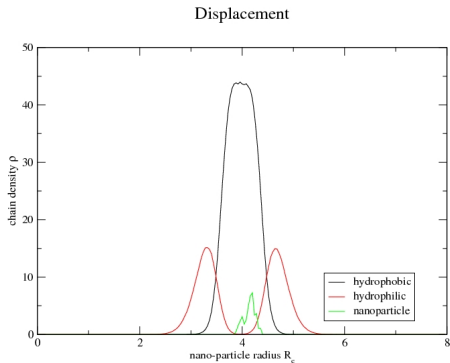
The hard-core radius and the Hamaker constant control the chemical potential.



We study sistematically the condition of stability of the nanoparticle in the hydrophobic shell of the membrane.

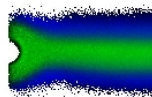
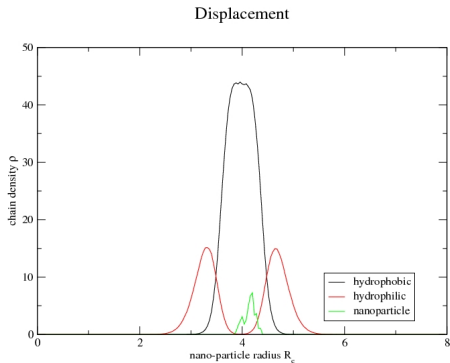


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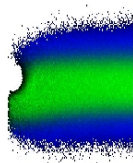
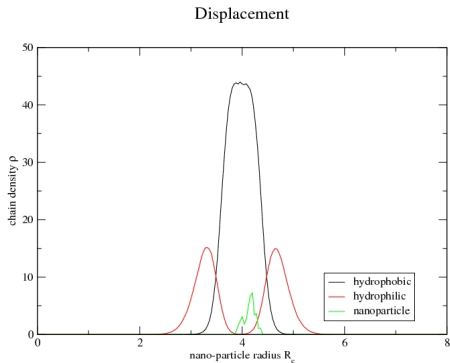
$$h = 2.0[k_B T], r = 4.5[nm]$$

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$$h = 1.0[k_B T], r = 13.1[nm]$$

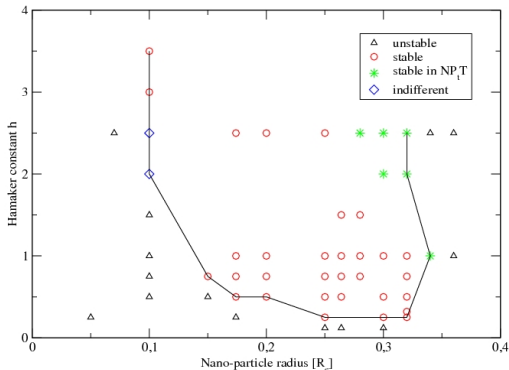
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$$h = 0.25[k_B T], r = 11.56[nm]$$

Different values of the Hamaker constant and the hard-core radius provide the stability of the nanoparticle.

Phase Diagram



The observed stable nanoparticles range within 3 and 9 [nm]. No information is given up to now on the contact angle.



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- **Improve the mapping with new experimental data.**

Dissipative Particle Dynamics<sup>1</sup>

- The forces are pair-wise interactions and conserve momentum  $\mathbf{F}_{ij}$

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Random Force

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Weighting function

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The membrane behaves like a incompressible fluid.

From the Gibbs's phase rule we can see how many degrees of freedom are required to describe the system

The most convenient way will be to define our model removing the solvent ( $c = 1$ ) and working in the coexistence line between the liquid and vapour interface ( $\phi = 1$ )

Removing the solvent we obtain a zero pressure sytem. The equation of state should therefore have a non zero density for vanishing pressure

A virial expansion of the equation of state can be approximate up to the third order

We define the following Hamiltonian

$$\frac{H}{k_B T} = \int v_{\alpha\beta} \rho^2(r) + v_{\alpha\beta\gamma} \rho^3(r) d^3 r$$

that conduce to the wanted equation of state

The membrane behaves like a incompressible fluid.

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$\alpha, \beta, \gamma = \{A, B\}$

$A$  hydrophobic,  $B$  hydrophilic

$$\rho_c + \frac{v_{aa}}{2} \rho_c^2 + \frac{v_{aaa}}{3} \rho_c^3 \stackrel{!}{=} 0 \Rightarrow v_{aa}$$

$$k_t^{-1} = -V(\partial_V P)_T = v_{aa} \rho + 2v_{aaa} \rho^2 + 1 \Rightarrow v_{aaa}$$

$$\chi \simeq \rho \left( v_{ab} - \frac{1}{2}(v_{aa} + v_{bb}) \right) \Rightarrow v_{ab}$$

$$\text{empirically: } v_{bb} = 0.1 \quad v_{bbb} = 0 \quad v_{aaa} = v_{aab} = v_{abb} = 0.95$$

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$$\text{Coarse-graining: } \rho \mapsto \frac{\rho R_e^3}{N m_{pc}} \quad v_{\alpha\beta} \mapsto \frac{v_{\alpha\beta} N^2}{R_e^3} \quad v_{\alpha\beta\gamma} \mapsto \frac{v_{\alpha\beta\gamma} N^3 m_{pc}}{R_e^6}$$

$m_{pc}$  monomers per chain,  $R_e$  end-to-end distance

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