# minimal models for lipid membranes: local modifications around fusion objects

Giovanni Marelli

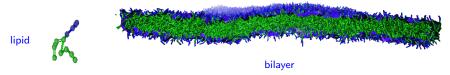
Georg August Universität, Göttingen

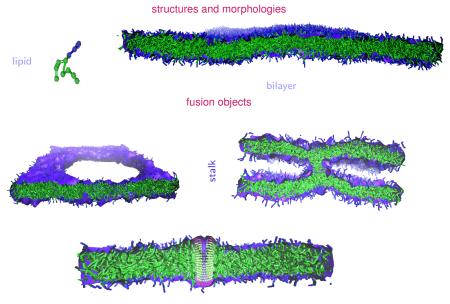
January 21, 2013

PhD defense









transmembrane protein

pore

# $membrane\ fusion$

# how does membrane fusion work?

viral fusion



K. K. Lee EMBO J. 29, 1299 (2010)

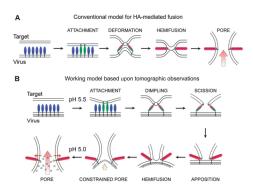
# membrane fusion

## how does membrane fusion work?

viral fusion

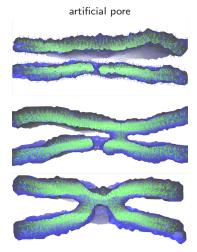
K. K. Lee EMBO J. 29, 1299 (2010)

#### possible (heuristic) explanations?



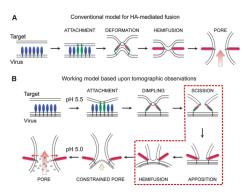
# membrane fusion

## how does membrane fusion work?



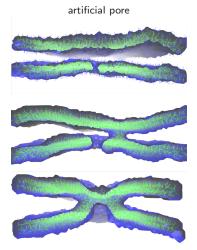
- capture morphology change and dynamics
- isolate metastable states

# possible (heuristic) explanations?



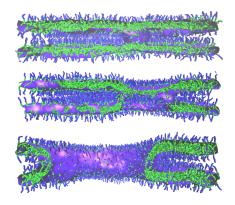
# membrane fusion

# how does membrane fusion work?



- capture morphology change and dynamics
- isolate metastable states

#### under lateral tension, low hydration

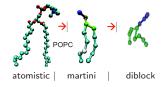


- identify the fusion pathways
- estimate energies and times
- role of the peptides

# contents

#### minimal models for collective phenomena in lipid membranes

lipids as diblocks



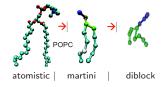
# coarse-grained solvent-free soft

- large time scales, many lipids
- computational efficiency
- sampling from thermal fluctuations

# contents

#### minimal models for collective phenomena in lipid membranes

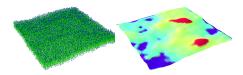
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# coarse-grained solvent-free soft

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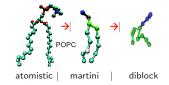
#### membranes as continuum sheets



# contents

# minimal models for collective phenomena in lipid membranes

#### lipids as diblocks



# coarse-grained solvent-free soft

- large time scales, many lipids
- computational efficiency
- sampling from thermal fluctuations

#### membranes as continuum sheets

# mechanical properties of unperturbed membranes:

- time and length scales
- energetic contributions

# local modifications by fusion objects

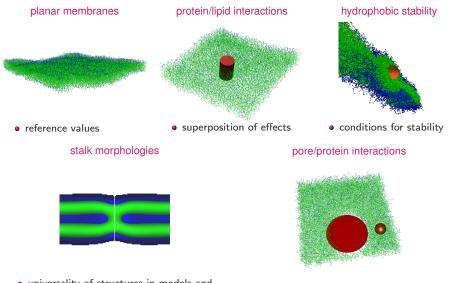
- thickness and density profiles in simulations and numerical calculations
- superposition of effects

#### lipid mediated interactions

- the peptides stabilize the pore
- the peptide lowers the line tension of the pore



# overview of the thesis

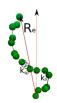


• universality of structures in models and experiments

superposition of thickness profiles

solvent-free, coarse-grained model

bonded (harmonic spring, bending):  $\frac{U_{sp}}{k_B T} = \sum_{i=1}^{N_b-1} \frac{k_{sp}}{2} (\mathbf{r}_i - \mathbf{r}_{i+1})^2$   $\frac{U_b}{k_B T} = \sum_{i=2}^{N_b-1} k_b (1 - \cos \theta_{i\pm 1})$ 

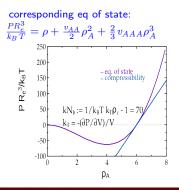


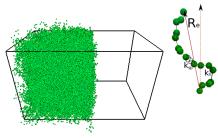
M. Hömberg and M. Müller, J. Chem. Phys. 132, 155104 (2010)

#### solvent-free, coarse-grained model

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non bonded (density functional):  $\frac{H_{nb}[\rho]}{k_B T} = \int \frac{\mathrm{d}^3 r}{R_c^3} \left( \frac{v_{AA}^2}{2} \rho_A^2 + \frac{v_{AAA}^3}{3} \rho_A^3 \right)$ 





(fluid/vapor phase coexistance)

the virial coefficients are set by  $\rho_{coex}$  and the  $kN_b$ :

$$v_{AA} = -2\frac{kN_b+3}{\rho_{coex}} \quad v_{AAA} = \frac{3}{2}\frac{kN_b+2}{\rho_{coex}^2}$$

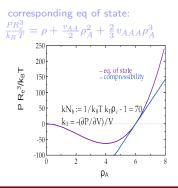
M. Hömberg and M. Müller, J. Chem. Phys. 132, 155104 (2010)

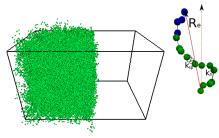
# model

#### solvent-free, coarse-grained model

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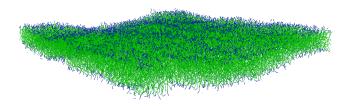
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 $\begin{array}{c} \underset{\chi N}{\text{mixed virial coefficient}} \\ \chi N \stackrel{MF}{=} \rho_{coex} \left( v_{AB} - \frac{1}{2} (v_{AA} + v_{BB}) \right) \end{array}$ 

M. Hömberg and M. Müller, J. Chem. Phys. 132, 155104 (2010)





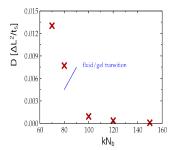
- time scale
- length scale
- coupling moduli
- fluctuation spectrum

# diffusivity

$$D = \lim_{t \to \infty} \frac{\left\langle \left( \mathbf{x}(t) - \mathbf{x}(0) \right)^2 \right\rangle}{4t}$$
  
timestep:  $1[\Delta t_s] \simeq 8[ps]$   
(martini 0.04[ps], atomistic 4[fs])

K. Weiß and J. Enderlein, ChemPhysChem 13, 990 (2012)

• varying  $kN_b$ 

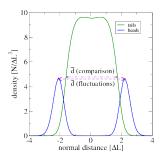


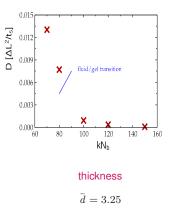
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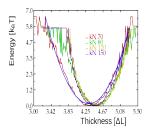
 $1[\Delta L] \simeq 1.08[nm]$ 

N. Kučerka et al. Biophysica Acta 1808, 2761 (2011)

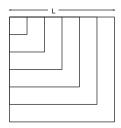
# we sample from thermal fluctuations

$$\Delta F_s = \int \mathrm{d}x \mathrm{d}y \, \frac{k_s}{2} \frac{(s(x,y) - \bar{s})^2}{\bar{s}^2}$$

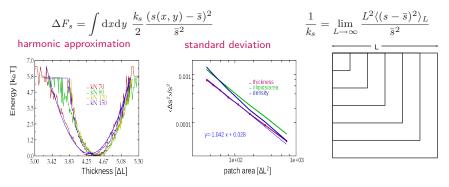
harmonic approximation



s fluctuating quantity,  $k_s$  coupling constant, L patch area.

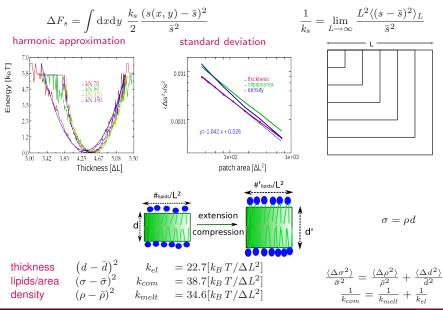


# we sample from thermal fluctuations



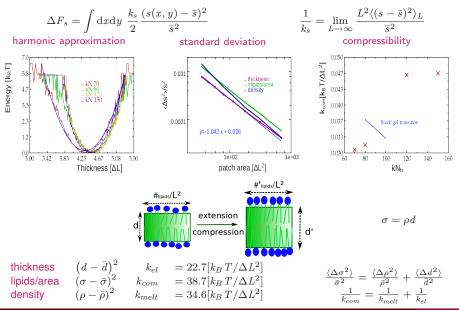


#### we sample from thermal fluctuations



minimal models for membranes

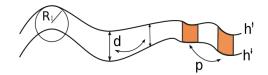
we sample from thermal fluctuations



minimal models for membranes

bending (height), peristaltic (thickness), protrusion (lipid separation)

$$F = \frac{1}{2} \int \mathrm{d}x \mathrm{d}y \, \left( k_{ben} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) + k_{el} \frac{(d-\bar{d})^2}{\bar{d}^2} + k_{melt} \frac{(\rho-\bar{\rho})^2}{\bar{\rho}^2} \right) \, \text{Helfrich model}$$



bending:  $k_{ben}$  bending rigidity peristaltic:  $k_{el}$  elastic coupling  $c_0$  spontaneous curva a area/lipids

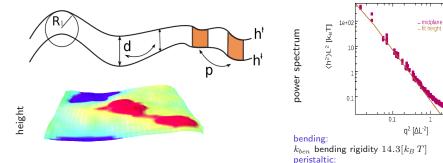
 $\zeta := c_0 - a \partial_a c_0$ 

#### protrusion

- $k_{\lambda}$  protrusion rigidity
- $\gamma_{\lambda}$  microscopic surface tension

bending (height), peristaltic (thickness), protrusion (lipid separation)

 $F_{ben} = \frac{1}{2} \int dx dy \ k_{ben} (\nabla^2 h(x, y))^2$  Monge representation



$$k_{el}$$
 elastic coupling

 $c_0$  spontaneous curvature a area/lipids  $\zeta := c_0 - a\partial_a c_0$ 

#### protrusion:

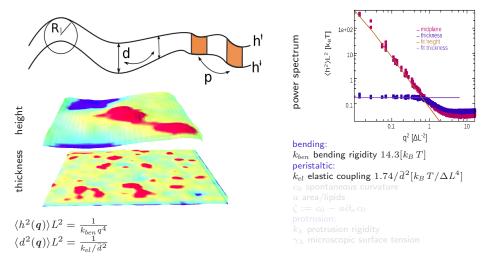
- $k_{\lambda}$  protrusion rigidity
- $\gamma_{\lambda}$  microscopic surface tension

$$\langle h^2({\it q}) \rangle L^2 = rac{1}{k_{ben} q^4}$$

10

bending (height), peristaltic (thickness), protrusion (lipid separation)

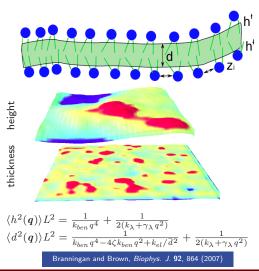
$$F_{ben} = \frac{1}{2} \int \mathrm{d}x \mathrm{d}y \ k_{ben} (\nabla^2 h(x, y))^2 \qquad F_{per} = \frac{1}{2} \int \mathrm{d}x \mathrm{d}y \ k_{el} \frac{d^2(x, y)}{\bar{d}^2}$$

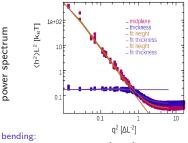


minimal models for membranes

# energetic contributions

#### bending (height), peristaltic (thickness), protrusion (lipid separation)





 $k_{ben}$  bending rigidity  $12.5[k_B T]$ peristaltic:  $k_{el}$  elastic coupling  $1.96/\overline{d}^2 [k_B T/\Delta L^4]$  $c_0$  spontaneous curvature

 $a \operatorname{area}/\operatorname{lipids}$ 

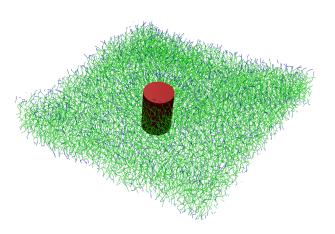
$$\zeta := c_0 - a\partial_a c_0 = 0.14/\bar{d}[\Delta L^2]$$
  
protrusion:

- $k_{\lambda}$  protrusion rigidity
- $\gamma_{\lambda}$  microscopic surface tension

we extract:  $k_{ben}$ ,  $\zeta$ ,  $k_{el}$ 

power spectrum

# hydrophobic inclusion



- radial thickness profile
- weakening of the membrane
- continuum model
- superposition of effects
- pore stabilisation

# thickness profile

# contact angle and thinning

cylindrical transmembrane protein

cluster of connected monomers

radius  $1[\Delta L]$ hydrophobic mismatch 1.3  $N_b/A = 38[\Delta L^{-2}]$ 



# thickness profile

# contact angle and thinning

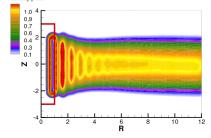
cylindrical transmembrane protein

cluster of connected monomers

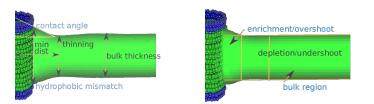
radius  $1[\Delta L]$ hydrophobic mismatch 1.3  $N_b/A = 38[\Delta L^{-2}]$  $\theta_c = 31^o$ 



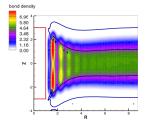
density phob



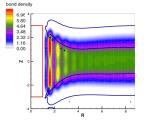
#### radial thickness profiles



# lipid frustration



bond length



oil partitions close to the peptide

oil density

2.17 1.81 1.45

bond length

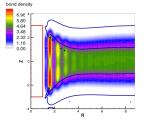


# oil molecule:

- dodecane
- hydrocarbon tail

density of the oil

- the oil loses its translational entropy
- acts as a sensor to partion where most of the stress is



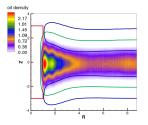
bond length



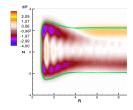
# oil molecule:

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# oil partitions close to the peptide



density of the oil

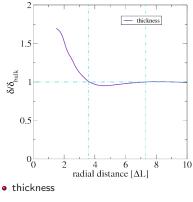


 point to point hydrophobic density difference with and without oil

- the oil loses its translational entropy
- acts as a sensor to partion where most of the stress is

# mean separation

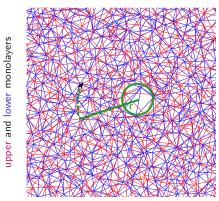
rescaled to the bulk value thickness from the polar/apolar density intersection



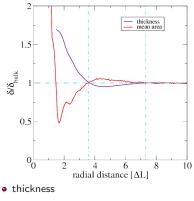
mean area

#### mean separation

rescaled to the bulk value thickness from the polar/apolar density intersection bead number normalization

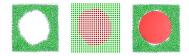


Delaunay triangulation



mean area

# lowering the line tension of the pore

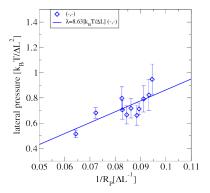


classical nucleation theory

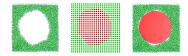
$$G = G_0 + 2\pi R_p \lambda - \pi R_p^2 P_l$$

line tension of the pore in  $NVT\$ ensemble

$$P_l = \frac{\lambda}{R_p}$$



# lowering the line tension of the pore



classical nucleation theory

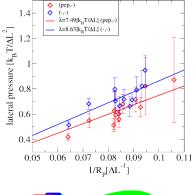
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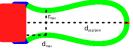
line tension of the pore in  $NVT\$ ensemble

$$P_l = \frac{\lambda}{R_p}$$

• the peptide lowers the line tension of the pore

"pore's radius correction"  $P_{l} = \frac{\lambda}{R_{p}} \left( 1 - \alpha_{pep} \frac{N_{pep}}{R_{p}} \right)$ 

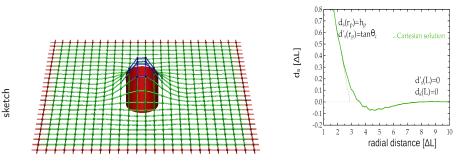




# continuum description

### elastic model

$$\begin{split} F &= \int \mathrm{d}x \mathrm{d}y \quad \frac{k_{ben}}{2} (\nabla^2 d_s)^2 + 4k_{ben} c_0 \nabla^2 d_s + 2\frac{k_{ben}\zeta}{d} d_s \nabla^2 d_s + \frac{k_{el}}{2d^2} d_s^2 \\ d_s(x,y) &:= d(x,y) - \bar{d} \quad \text{Euler Lagrange equations:} \quad \frac{\delta F}{\delta d_s} = 0 \\ \left(k_{ben} \nabla_c^4 + \frac{4k_{ben}\zeta}{d} \nabla_c^2 + \frac{k_{el}}{d^2}\right) d_s(x,y) = 0 \quad \nabla_c^2 := \begin{pmatrix} \partial_x^2 & 0 \\ 0 & \partial_y^2 \end{pmatrix} \end{split}$$

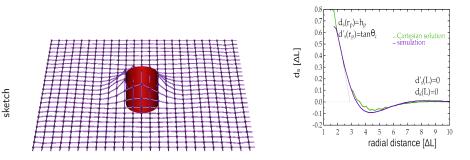


B. West, F. L. H. Brown and F. Schmid, Biophys. J. 96, 101 (2009)

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B. West, F. L. H. Brown and F. Schmid, Biophys. J. 96, 101 (2009)

minimal models for mem

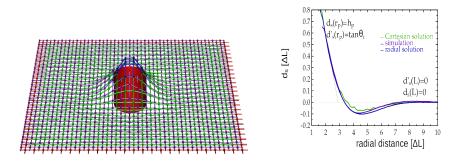
lipids/peptide

# continuum description

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 $\left(k_{ben}\nabla_r^4 + \frac{4k_{ben}\zeta}{d}\nabla_r^2 + \frac{k_{el}}{d^2}\right)d_s(r) = 0 \qquad \nabla_r^2 := \frac{1}{r}\partial_r(r\partial_r)$ 

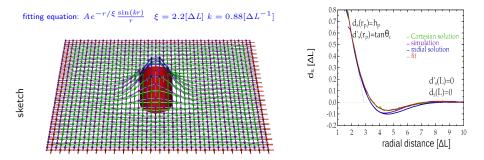


B. West, F. L. H. Brown and F. Schmid, Biophys. J. 96, 101 (2009)

#### elastic model

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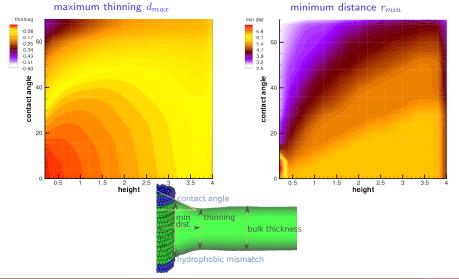


B. West, F. L. H. Brown and F. Schmid, Biophys. J. 96, 101 (2009)

# fine exploration

#### thickness characteristics

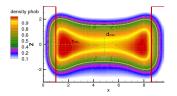
the solution of the continuum model is  $10\,000$  times faster than simulations

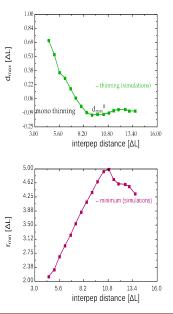


inimal models for membranes

# superposition of effects

### two inclusions





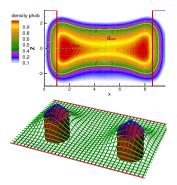
- $r_{min}^{II}$ : minimum splits in two
- $d_{max}^{II}$ : superposition effect is stronger

clusterisation of gramicidins: (hydrophobic matching)

T. Harroun et al Biophys. J. 76, 937 (1999)

# superposition of effects

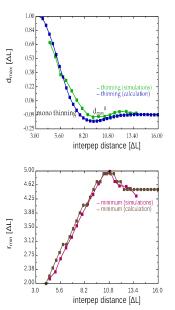
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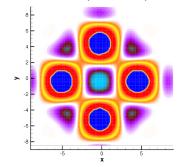


# many peptides

8 peptides 6 thinning thickness 0.37 > 0 0.22 -2 0.08 -0.07 -0.21 -0.36 -0.50 0 X 5 -5

# thickness (simulations)

# thickness (calculations)



# many peptides

thickness 0.37

0.22

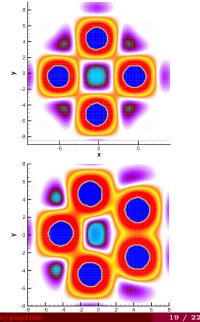
0.08

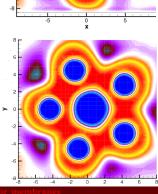
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8 peptides 6 thinning > ( -2 -5 0 x

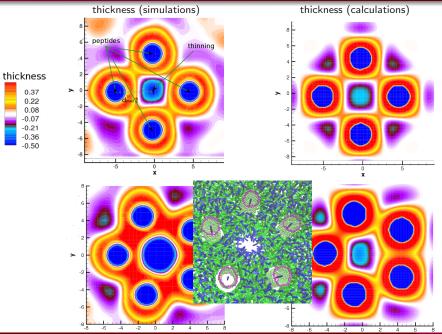
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thickness (calculations)





# many peptides



#### summary

### unperturbed membranes

#### • sampling from thermal fluctuations

- time and length scale
- energetic contributions

#### opwer spectrum

- material properties
- parametrisation of the continuum model

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### lipid/peptide

- thickness profile
  - undershoot below the bulk value
  - good agreement between simulations and calculations

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- at the interface with the peptide (pore's rim and stalk's ends)
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#### minimal models

- fast and fine exploration of phenomena
- tuned by more detailed models
- confirm the multiscale approach (universality of stalk's morphologies) (bilayer repulsion)

### lipid/peptide

- dynamics
  - study the peptide diffusion by changing the peptide description
  - study the lipid diffusivity depending on the radial distance from the peptide

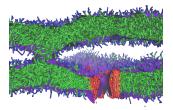
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#### stalk/pore/peptides

- competing line tensions
  - interactions between a boundle of peptide and a stalk



thank you for your attention

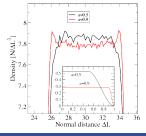
acknowledgments Marcus Müller Yuliya Smirnova Jelger Risselada Martin Hömberg Kostas Daoulas Ulrich Welling



### simulation technique: MD + MDPD thermostat

#### characteristic:

- the total force is the sum of the conservative, random and dissipative forces  $\mathbf{F}^{tot} = \mathbf{F}^c + \mathbf{F}^r + \mathbf{F}^d$
- the forces are pairwise  $\mathbf{F}_{i}^{tot} = \sum_{j} \mathbf{F}_{ij}^{c} + \mathbf{F}_{ij}^{r} + \mathbf{F}_{ij}^{d}$
- the forces are soften via a weighting function  $\mathbf{F}_{ij} = \mathbf{F}_{ij} \, w(r_{ij})$



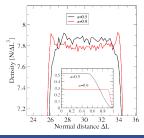
P. Español and P. B. Warren Europhys. Lett. 30 191 (1995)

- local density dependance
- "pragmatic extension of the classical DPD that allows one to prescribe the thermodynamic behavior of a system"
- NPT
- *NVT*
- $NP_t T$  tensionless membrane.

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### features of the multibody DPD:

- local density dependance
- "pragmatic extension of the classical DPD that allows one to prescribe the thermodynamic behavior of a system"

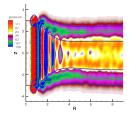
### ensembles

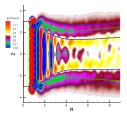
- *NPT*
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# oil relaxation

$$P_{\alpha\beta}(r,z) = k_B T \rho(r,z) + k_B T \rho(r,z) + \frac{1}{V} \sum_{ij}^{(r,z)} \frac{\mathrm{d} U_{ij}}{\mathrm{d}r} \frac{x_{ij}^{\alpha} x_{ij}^{\beta}}{|x|}$$

normal component  $P_z^l(r,z) = P_{zz}(r,z) - \frac{P_{\theta\theta}(r,z) + P_{rr}(r,z)}{2}$  no oil with oil



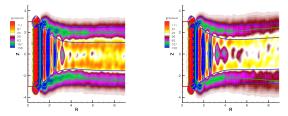


- identification of the interface
- position of the relaxation by the oil

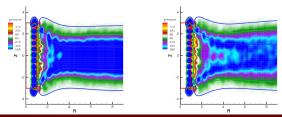
#### minimal models for membranes

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radial component  $P_r^l(r,z) = P_{rr}(r,z) - \frac{P_{\theta\theta}(r,z) + P_{zz}(r,z)}{2}$ 



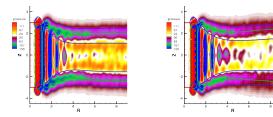
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minimal models for membranes

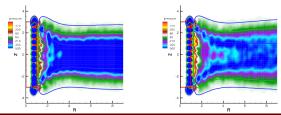
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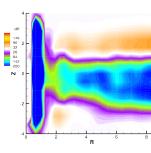
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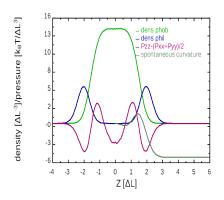
### lateral pressure difference



- identification of the interface
- position of the relaxation by the oil

minimal models for membranes

### calculation of the spontaneous curvature



lateral pressure

$$P_{zz}^{l}(r,z) = P_{zz}(r,z) - \left(\frac{P_{xx}(r,z) + P_{yy}(r,z)}{2}\right)$$

spontaneous curvature

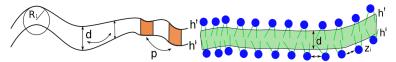
$$k_{ben} c_0 = \int_0^L \mathrm{d}z \, z P_{zz}^l(z)$$

 $c_0 = -0.36[\Delta L^{-1}]$ 

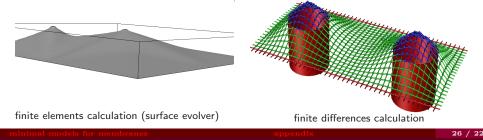
normal lateral pressure profile

# elastic model

$$F = \int dx dy \left( k_{ben} 2(\nabla^2 h)^2 + k_{\lambda} (z^{+2} + z^{-2}) \right) + \gamma_{\lambda} \left( (\nabla z^+)^2 + (\nabla z^-)^2 + \nabla h \nabla z^+ / 2 + \nabla d \nabla z^-) \right) + \frac{k_{com}}{2} d^2 + k_{ben} c_0 \nabla^2 d + \frac{k_{ben} \zeta d}{d} \nabla^2 d + \frac{k_{ben}}{2} (\nabla^2 d)^2 \right)$$

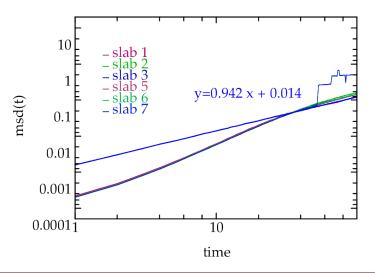


comparison with the solution of the differential equation in 2d

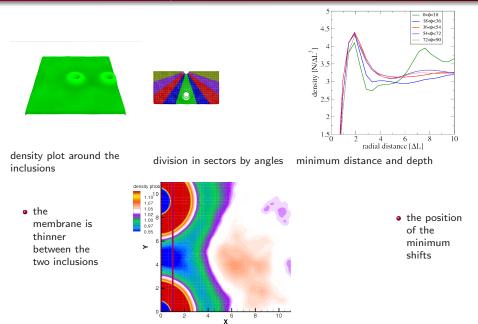


# slab mean square displacement

mean square displacement in radial slabs

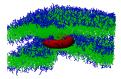


# superposition of effects/two inclusions

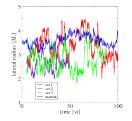


# stiffness of a circular stalk

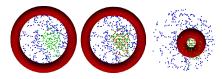
we search iteratively a torus surrounding the stalk with:



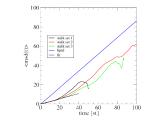
- the smallest lateral radius
- the largest normal radius
- the most contact with heads
- the less contact with tails



#### we count the beads inside the torus



- the center of mass of the beads is the torus position
- the radius of the torus is  $r_t = \sqrt{A}/2\pi$ , A the area occupied by the beads



 $D_{stalk} = D_{lipid}/3$ 

radial evolution

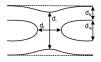
diffusivity

## stiffness and density profile

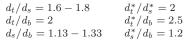
exp data PhD thesis of S. Aeffner

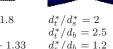
radial density profiles - model/scale dependence Martini force field, dft, implicit solvent explicit solvent

flexible chains, explicit solvent











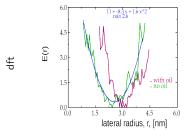
 $d_t^*/d_s^* = 2.1$  $d_t^*/d_b = 2.1$  $d_{e}^{*}/d_{b} = 1.0$ 



 $d_t^*/d_s^* = 2.9$  $d_t^*/d_b = 2.2$  $d_{e}^{*}/d_{h} = 0.77$ 

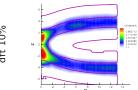


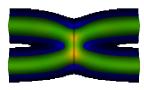
Martini



94 + -74 x + 15 x^ min 2.5 3.4 Ē - with oil 1.0 1.9 2.8 3.6 4.5 lateral radius, rtor [nm]

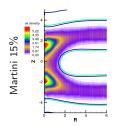
# addition of small oil chains in the bilayers



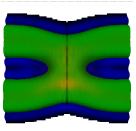


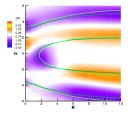


### oil partitioning



### density profile





# tails density difference

